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Distribution of joints: probabilistic modelling and case study near Cardiff (Wales, U.K.)

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Abstract—In this paper a new probabilistic two-dimensional model for simulating joint development in a single vertical set cutting horizontally layered rocks is presented. The problem of considering the probability of joints nucleating within limestone beds and the probability of joints propagating across mudstone interbeds is solved using matrix analysis. Sources of dispersion, relationships between joint spacing and bed thickness, and the influence of pre-existing joints on vertical propagation were taken into account. Simple assumptions were made concerning the sense of vertical joint propagation, poorly constrained by field data in most cases. We demonstrate that, with the input of just a few geometrical and statistical parameters, realistic cross-sections can be constructed based on the probabilistic modelling. Our field analysis aimed to: (1) provide real values for the statistical parameters; and (2) compare actual and simulated patterns in order to check our results. Input parameters include bed thicknesses, average numbers of joints and the proportion of joints cutting two adjacent limestone beds. Field data were collected from coastal exposures in Liassic rocks at Llantwit Major (Wales, U.K.). At the sample sites, tabular layers comprise alternating decimetric limestones and centimetric mudstone interbeds. They are cut by two orthogonal sets of vertical joints. We studied the distribution of the dominant joint set striking N170°. Both the models and the outcrops display similar characteristics, e.g. a log-normal distribution of joint spacing and a negative exponential distribution of the number of joints vs the number of beds crossed. Our probabilistic modelling is applicable to other situations with a limited number of numerical constraints obtained from field or sub-surface observations. © 1997 Elsevier Science Ltd.

INTRODUCTION

During the 1990s the development of natural joints was the focus of many studies (Narr and Suppe, 1991; Rawnsley *et al.*, 1992; Rives, 1992; Gross, 1993; Mandal *et al.*, 1994; Gross *et al.*, 1995). In this paper we take the opportunity of comparing a comprehensive set of field observations on joints with the results of probabilistic modelling. Our study site is located in Liassic (Lower Jurassic) rocks exposed at Llantwit Major near Cardiff (Wales, U.K.), where excellent exposures along the cliffs of the Bristol Channel allow the detailed observation of vertical sets of joints cutting horizontally layered sedimentary rocks (alternating limestones and mudstones). In particular, we collected data on joint spacing, the number of joints per bed, the thicknesses of beds and the vertical continuity of joints at bed interfaces. We also measured data on the orientations of the joints.

The main purpose of this paper is not to describe or interpret in detail an actual pattern of joints, but to present a new probabilistic approach allowing reconstruction of some statistical characteristics of such a pattern with a small number of parameters. The accurate knowledge of a real pattern of joints is, however, indispensable because it enables us to check that our model provides acceptable simulated sections. The description of the main system of joints of Llantwit Major, referred to in this paper, is principally used for this comparative check, rather than for constraining the model in detail.

Although the classification of tectonic joints remains a controversial subject (Hancock, 1985; Pollard and Aydin, 1988), we do not attempt to address this problem but rather we aim at understanding spatial relationships between individual joints within a single set. Because most of the structures that we observed in this set are barren fractures interpreted as being initiated as extension fractures and propagated as mode 1 cracks, they would be called joints by all workers irrespective of their views about other fracture classes. We also aim to statistically simulate these relationships via a simple numerical model based on consideration of probabilities discussed below. With computational techniques based on the probabilistic approach, we create a synthetic joint network that should account for the major geometrical characteristics of the real one. The model is refined by general statistical parameters arising from the field observations.

The probabilistic modelling of fractures and joints has already been discussed by Priest and Hudson (1976), Dershowitz (1984) and Chiles (1989). Our probabilistic two-dimensional model is based on considering probabilities of fracture initiation within limestone beds, as well as the probability of their propagating across mudstone beds (Helgeson and Aydin, 1991). Modelling fracture patterns in a rock mass (e.g. for a fractured reservoir) is a three-dimensional problem. However, we address the very common, albeit particular, case of parallel vertical joints perpendicular to tabular layered rocks exposed in a vertical section, which trends perpendicular to the strike of the studied joint set.

A major problem in modelling joint distributions is the large number of degrees of freedom (e.g. rheology, distribution of pre-existing discontinuities, pore pressure, stress field), which in the absence of relevant field data does not allow the model to be tightly constrained. Thus, before model conception, it is essential to select an example of an actual joint distribution that can be used to constrain realistic distributions.

TECTONIC SETTING AND STRUCTURAL HISTORY OF THE LLANTWIT MAJOR AREA, SOUTH WALES

The study area lies on the northern (Welsh) margin of the East Bristol Channel Basin (EBCB), a sub-basin within the Bristol Channel Basin, a depocentre containing up to 3400 m of Mesozoic–Cenozoic sediments (Kammerling, 1979). Brooks *et al.* (1988) consider the western part of the EBCB to be a half-graben, bounded by the down-to-the-south Bristol Channel fault zone off the Welsh coast. Onshore, the Mesozoic rocks of the basin overstep and onlap on Palaeozoic rocks deformed during the Variscan orogeny (Owen and Weaver, 1983). The most detailed appraisal of the tectonic history of the Bristol Channel Basin is that of Nemcock *et al.* (1995) whose field data were mainly collected along the Welsh coast. They recognized four tectonic phases which are outlined below.

(1) NW–SE extension during Permo-Triassic rifting initiated faulting and in South Wales resulted in the formation of NE-striking neptunian dykes and extensional veins.

(2) Triassic–Aptian rifting permitted basin subsidence to continue but as a consequence of NE–SW to NNE–SSW extension reactivating Variscan faults and forming new ones.

(3) The Late Cretaceous was a time of tectonic quiescence during which faults were neither reactivated nor formed. Transgression and onlap of Late Cretaceous sediments (mainly chalk) occurred in the offshore sector of the EBCB.

(4) Positive inversion of the EBCB started in the Palaeocene, reached a climax during the Oligocene and continuing into the Early Miocene. Nemcock *et al.* (1995) concluded that their palaeostress reconstructions for this phase are broadly in accord with those of Bergerat (1987), that is, N–S compression in the late Eocene, succeeded by NE–SW compression in the Early Miocene. The principal structures achieving inversion were thrusts and related strike-slip faults, some of which were neoformed but many of which were inherited Mesozoic normal faults. Whether or not the NW–SE late Miocene–Recent compression documented by Bevan and Hancock (1986) and Bergerat and Vandyke (1994), from localities further to the east and south, affected the EBCB is not known.

Most studies of outcrop-scale structures in the Early Mesozoic rocks on both the Welsh and English shores of the Bristol Channel have focused on faults but on the Welsh side the superbly exposed joints have also attracted attention (Roberts, 1974, 1995; Rawnsley *et al.*, 1992; Rives, 1992; Rives *et al.*, 1992). The joints at Llantwit Major cut beds within the 'Blue Lias', or Porthkerry Formation, of Hettangian–Sinemurian age. This sequence comprises tabular layers of decimetric-thick micritic limestones, of diagenetic origin according to Wobber (1965), and centimetric-thick mudstone interbeds.

At Llantwit Major (SS 955 675, Figs 1 & 2), a dominant set of joints strikes roughly N170° and strikes at roughly right angles to the trend of the cliff (Roberts, 1974, 1995; this study). At Nash Point (SS 915 683, Fig. 1), to the west of Llantwit Major, the strike of the dominant set is also N170° but at Lavernock Point (ST 187 682, Fig. 1), to the east of Llantwit Major, it departs slightly from this trend striking N160°. No evidence of shear was found within these main regional set of joints. The favoured explanation for the presence of the N170° vertical joints is that they formed in response to a Late Cretaceous–Early Miocene regional stress field (corresponding to the inversion of the EBCB) in which the least effective principal stress (σ_3) was horizontal and oriented approximately N080°, perpendicular to the strike of the joint set. Based on abutting relationships, the N170° joint set contains the oldest joints among those cutting the Liassic limestones. Although less important, other joint sets are present and, generally, members of these sets are less planar and shorter than those in the main set. Note that the joint set striking N075° at Nash Point and Llantwit Major was interpreted by Caputo (1995) as being coeval with the N170° one.

In addition to joints, other small fractures cutting the Liassic limestones include microveins less than 1 mm wide and veins ranging in width from 0.5 to 5.0 cm. Microveins, many of which are significantly less than 1 mm wide, belong to several sets, which developed early

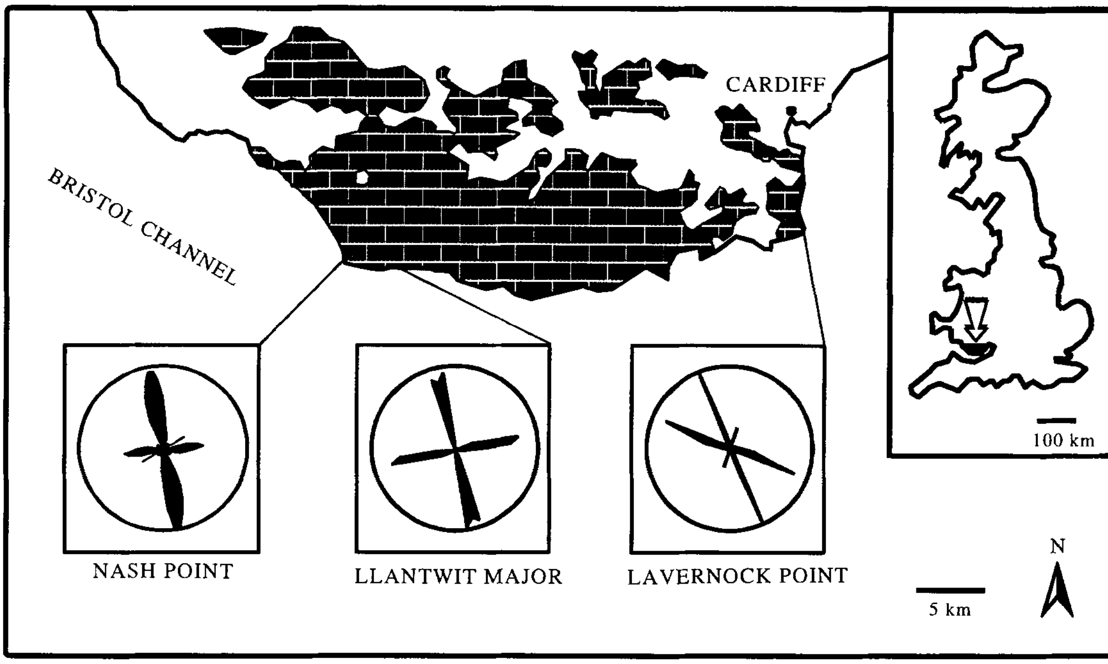


Fig. 1. Map of the outcrop of Liassic rocks on the southern coast of Wales with frequency diagrams of joint set azimuths. Data are from Lavernock Point (ST 187 682), Llantwit Major (SS 955 675) and Nash Point (SS 915 683).

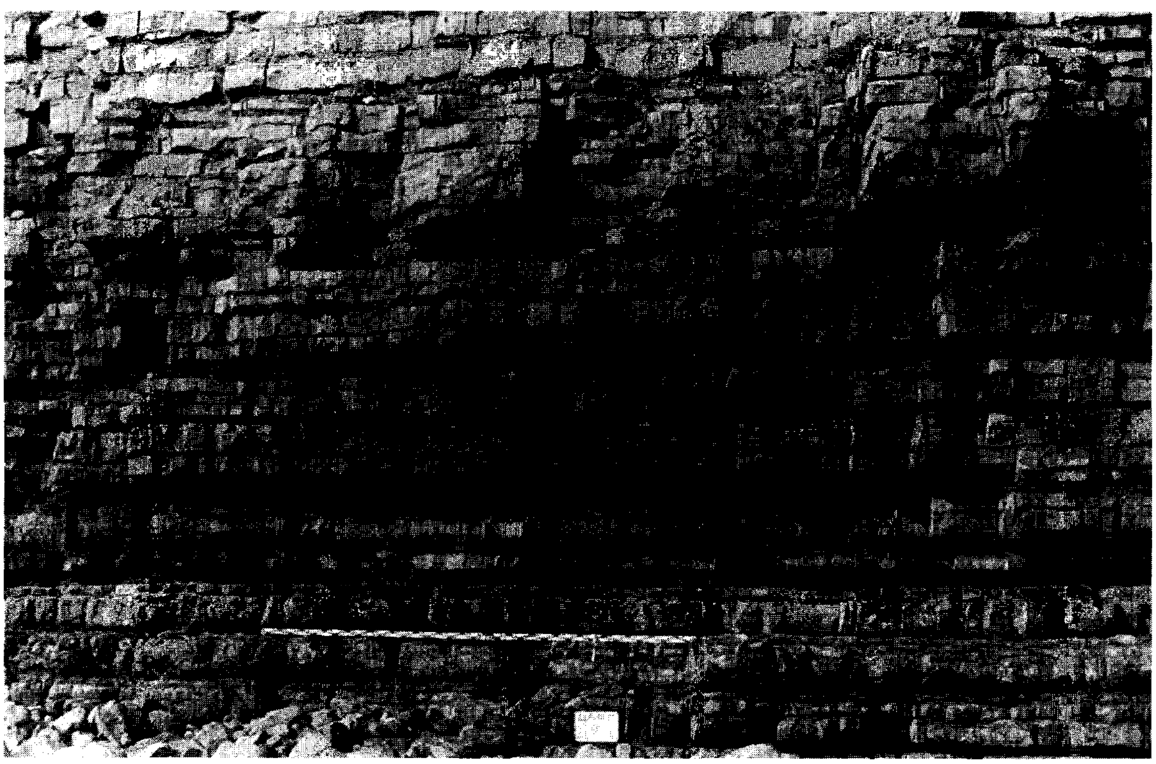


Fig. 2. Studied outcrop at Llantwit Major (SS 955 675). View to the north; scale bar = 4 m.

in the brittle history of the rocks, because they are cut by other structures and were not reactivated during later fracturing events. Some of the wider mineral veins are sealed cracks, but others display axial discontinuities. The majority of these veins are related to geometrically imposed accommodation effects adjacent to faults, and thus their spacing is controlled by the distribution of faults rather than by bed thicknesses.

Although it is reasonable to assume that the N170° set is related to Late Cretaceous–Early Miocene compression (with σ_1 horizontal and parallel to the joint planes, and σ_3 also horizontal but perpendicular to the joint planes), the origin of the other sets is more controversial. For example, the second major set, which strikes N110°, was attributed by Rives (1992) to local accommodation effects at Lavernock Point (Fig. 1). He attributed the development of the set to local deformation within the Penarth anticline.

PRINCIPLE OF PROBABILISTIC MODELLING

Previous probabilistic models of joint distribution are based on the use of statistical laws, particularly on the use of the Poisson law. In this paper, we adopt a probabilistic approach based on the probabilities of nucleating joints within limestones as well as propagating through mudstones and into neighbouring limestones.

As is well known, joint spacing is controlled by many factors. For instance, there is a relationship between joint spacing and the presence or absence of neighbouring faults, with joints being more abundant near faults (even if the ultimate mechanism controlling joint spacing remains the stress shadow, see Gross *et al.*, 1995). For this reason, we considered in our study only sections which do not include faults and are contained within otherwise undeformed rock volumes. In such settings, simple observation reveals, and data collection confirms, that the thickness of a bed influences the spacing of the joints cutting the bed (Narr and Suppe, 1991; Gross, 1993; Gross *et al.*, 1995). In fact, bed thickness controls joint height and then the stress shadow, which is proportional to joint height, controls joint spacing. Furthermore, bedding surfaces bounding layers of contrasting mechanical properties also act as barriers to the vertical extent of many joints in the sections we studied.

As a consequence, we attempt to construct a probabilistic model of joint distribution which is essentially based on: (1) the correlation between bed thickness and spacing; and (2) the control exerted by bedding surfaces on the vertical propagation of joints (which is implicit in calculating probabilities of propagation). Because we are dealing with a single set of joints cutting uniformly dipping rocks, direct comparisons between beds of various thickness can be made.

Our model is a two-dimensional probabilistic one, which takes into account as parameters some of the major average characteristics of our data, and not the detailed geometry of the pattern of joints. We thus consider, in section, planar and parallel joints perpendicular to the section and to 'perfect' layers (homogeneous, horizontal and consistent in width) of alternating limestone and mudstone.

Our model is also 'disconnected', which means that it does not receive any joints propagating from outside the volume being considered. One may thus expect different statistics for the top and bottom beds, compared to the middle beds in the model, because of the absence of joints propagating from outside into the volume simulated. This is not the case in our model because the number of joints inside it depends on average values observed for each bed, so that the absence of inward-propagating joints at the model boundaries is compensated for by extra joints nucleating inside the simulated outcrop.

In more detail, the joints in a given bed either nucleate in the bed or have propagated from the top and bottom. The proportion that have propagated can be estimated from direct observation at the outcrop, by counting the number of joints which pass across the bed boundaries. Because the model is constrained by average numbers of joints in beds, modifying the propagation probabilities does not affect the number of simulated joints in a given bed. We assume that a joint can occur in any of the limestone beds, according to the probability law discussed below (Fig. 3a–c). Furthermore, we assume that a joint can propagate through a mudstone layer from a limestone bed towards another limestone bed. Note that mudstone beds are only considered as mechanical boundaries for limestone beds in our analysis model, according to a second probability law which is also discussed.

The mathematical analysis requires the following input data:

(1) m, the number of limestone beds;
(2) n_T, the number of joints in the outcrop;
(3) $n_J(i)$, the number of joints in limestone bed i with $1 \leq i \leq m$ counting downwards through the sequence.

The first problem is determining the probability, $P_P(i, j)$, of each joint propagating from the limestone bed i to the limestone bed j with $j = i + 1$ or $j = i - 1$ (Fig. 3d). The direction of propagation of most joints was dominantly horizontal, according to the observations made by Roberts (1995) near Lavernock Point, where he stated that nucleation was generally near the base of beds. Because bed boundaries are discontinuities across which the elastic crack-tip stress field is not transmitted, propagation is usually contained in one bed. If we assume that joints propagated via elliptical cracks, they would abut first the top or bottom of a bed depending on the location of the nucleation point. Because nucleation points of plumes are mainly sited in limestones we infer that joints formed first in limestones.

In our modelling, we arbitrarily assume that 50% of joints crossing mudstone beds propagated downwards and 50% propagated upwards. This assumption requires careful examination. In our field analysis of the actual joint pattern we could not easily determine the proportion of joints propagating upwards or downwards across bed boundaries. In other settings it has been shown that this proportion may differ from half-and-half (e.g. Engelder *et al.*, in press) which may fit some mechanical requirements, such as for the relaxation of overburden stress in the upward direction. However, we decided to adopt the simplest statistical rule because adopting an asymmetric rule in the probabilistic model would have biased the calculation of the nucleation probabilities

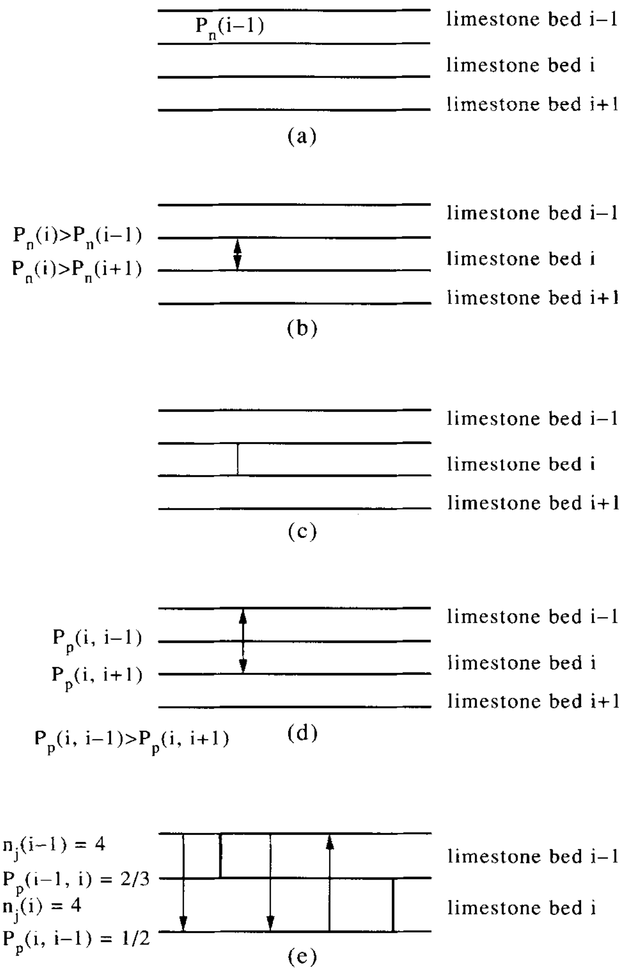


Fig. 3. Model principles (mudstone interfaces are omitted). (a) Probabilities for a joint to nucleate in limestone beds $i-1$, i and $i+1$. (b) Joint nucleation is expected to occur in limestone bed i, according to nucleation probability for limestone bed i ($P_n(i)$), better than nucleation probabilities for adjacent limestone beds $i-1$ and $i+1$ ($P_n(i-1)$ and $P_n(i+1)$, respectively). (c) Vertical propagation of the nucleated joint to upper and lower boundaries of limestone bed i. (d) Transition probabilities from limestone bed i to adjacent limestone beds $i-1$ and $i+1$. The nucleated joint is extended to limestone bed $i-1$, according to $P_p(i, i-1) > P_p(i, i+1)$. (e) Transition probabilities between limestone beds $i-1$ and i. The number of joints present in limestone bed i is given by the value $n_J(i) = 4$, two joints come from the upper limestone bed $i-1$ and two joints are nucleated inside this bed, one of them extended to adjacent limestone bed $i-1$, so the transition probability for a joint to extend from i to $i-1$ is given by $P_p(i, i-1) = 1/2$.

depending on the position of the layer in the model. Because we use a linear system in our mathematical analysis, introducing contrasting assumptions for upward and downward propagation would have affected dramatically the spatial periodicity of joints. For instance, with 100% of joints propagating upwards, the distribution of joints in the upper beds in the model would be significantly perturbed and would no longer fit the rather regular observed distribution. Considering the mathematical constraints in our analysis, this assumption has no major consequence as far as the main aim of our modelling remains a realistic simulation of the main geometrical properties of the joint pattern.

Simple field observation provided direct evidence of the proportion of joints which stopped at, or cut through, bedding planes (and adjacent mudstones). As a consequence the propagation probabilities were easy to define, based on ratios between numbers of joints crossing bedding planes and the total number of joints present at a limestone bed boundary (Fig. 3e). The determination of the propagation probabilities can be carried out by counting $n_t(i, i+1)$, the number of joints common to successive limestone beds i and $i+1$, with $1 \leq i \leq m-1$.

We define $P_n(i)$ as the unknown probability of a joint nucleating in the limestone bed i. This probability is unknown because for joints cutting more than one layer, it is generally impossible, from observation, to distinguish in a given bed between the joints which nucleated from inside the bed and those which propagated from outside it.

As a consequence of these definitions, for the limestone bed i, the number of joints present inside the bed, $n_J(i)$, is given by the addition of the following quantities.

(1) The number of joints nucleated within the limestone bed i, already defined as $P_n(i)n_T$.

(2) The number of joints nucleated inside all limestone beds located above limestone bed i in the studied pile, which could propagate into bed i. The number of these joints is obtained through the following summation. The number of joints nucleated inside limestone bed $i-1$ which could propagate to limestone bed i is given by $(P_n(i-1)n_T)\,P_p(i-1, i)$. The total number of joints which nucleated inside limestone beds $i-1$ to 1 and could propagate into limestone bed i is thus given by the following sum of products:

$$\sum_{q=1}^{q=i-1}\left\{P_n(q)n_T \prod_{r=q}^{r=i-1} P_p(r, r+1)\right\} = (P_n(1)n_T)\,P_p(1, 2)\ldots P_p(i-1, i) + \ldots + (P_n(i-1)n_T)\,P_p(i-1, i), \tag{1}$$

where q and r are simple dummy indices used in sums and products with $1 \leq q \leq i-1$ and $q \leq r \leq i-1$.

(3) The number of joints nucleated inside all the limestone beds located below limestone bed i which could propagate into it. This value is also obtained through a summation. The number of joints nucleated inside limestone bed $i+1$ which could propagate to limestone bed i is given by $(P_n(i+1)n_T)\,P_p(i+1, i)$. The total number of joints which nucleated inside limestone beds $i+1$ to m and could propagate to limestone bed i is thus given by:

$$\sum_{q=i+1}^{q=m}\left\{P_n(q)n_T \prod_{r=q}^{r=i+1} P_p(r, r-1)\right\} = (P_n(i+1)n_T)\,P_p(i+1, i) + \ldots + (P_n(m)n_T)\,P_p(m, m-1)\ldots P_p(i+1, i). \tag{2}$$

The total number of joints contained in limestone bed i can consequently be obtained as the sum of the three quantities given above, that is:

$$n_J(i) = \sum_{q=1}^{q=i-1} \left\{ P_n(q) n_T \prod_{r=q}^{r=i-1} P_p(r,\ r+1) \right\} + P_n(i) n_T + \sum_{q=i+1}^{q=m} \left\{ P_n(q) n_T \prod_{r=q}^{r=i+1} P_p(r,\ r-1) \right\}. \quad (3)$$

Expressing the sums and products, we obtain the following equations for beds 1 to m:

$$n_J(1) = (P_n(1) + P_n(2)\, P_p(2,\ 1) + \ldots + P_n(m)\, P_p(m,\ m-1) \ldots P_p(2,\ 1))\, n_T$$

....

$$n_J(i) = (P_n(1)\, P_p(1,\ 2) \ldots P_p(i-1,\ i) + \ldots + P_n(i) + \ldots + P_n(m)\, P_p(m,\ m-1) \ldots P_p(i+1,\ i))\, n_T$$

....

$$n_J(m) = (P_n(1)\, P_p(1,\ 2) \ldots P_p(m-1,\ m) + \ldots + P_n(m))\, n_T.$$

This system of equations is a linear one with m equations, where the m unknowns are the values of $P_n(i)$, with $1 \leq i \leq m$. The probabilities of joints nucleating in limestone beds can be determined provided that all the propagation probabilities, $P_p(i+1,\ i)$, with $1 \leq i \leq m-1$, and $P_p(j-1,\ j)$, with $2 \leq j \leq m$, are known. Likewise the number of joints in each limestone bed, $n_J(i)$, is given by field observation.

In order to solve this system of equations we consider the value $r(i) = n_J(i)/n_T$, with $1 \leq i \leq m$, which is the ratio between the number of joints inside limestone bed i and the number of joints inside the whole studied outcrop. Substituting $r(i)n_T$ for $n_J(i)$ in the linear system, we obtain the matrix equation $R = AV$ (equation (4)), with the vectors R (known) and V (unknown) and the matrix A (known) defined as follows:

$$R = \begin{pmatrix} r(1) \\ \cdot \\ \cdot \\ \cdot \\ n(m) \end{pmatrix} \qquad V = \begin{pmatrix} P_n(1) \\ \cdot \\ \cdot \\ \cdot \\ P_n(m) \end{pmatrix} \quad (4)$$

A =

$$\begin{bmatrix} 1 & P_p(2,1) & \ldots & \ldots & \prod_{j=1}^{j=m-1} P_p(j+1,j) \\ P_p(1,2) & 1 & \ldots & \ldots & \ldots \\ \ldots & \ldots & 1 & \ldots & \ldots \\ \ldots & \ldots & \ldots & 1 & P_p(m,m-1) \\ \prod_{j=1}^{j=m-1} P_p(j,j+1) & \ldots & \ldots & P_p(m-1,m) & 1 \end{bmatrix}$$

Table 1. Artificial input parameters for modelling (see also Fig. 4): $m = 5$ (number of limestone beds); $L = L' = 5$ m (length of the outcrop and length of the modelling outcrop); $n_T = 77$ (total number of joints belonging to the analysed set in the outcrop); $n_J(i)$ (number of joints in limestone bed i); $n_t(i,\ i+1)$ (number of joints common to limestone beds i and $i+1$); $e(i)$ (thickness of limestone bed i); $e'(i)$ (thickness of mudstone bed i)

Bed reference number	Limestones $e(i)$ (cm)	Limestones $n_J(i)$	Limestones $n_t(i,\ i+1)$	Mudstones $e'(i)$ (cm)
1	4.0	13	6	3.8
2	0.9	49	17	0.2
3	2.4	19	9	1.1
4	2.0	23	9	0.3
5	2.9	15	—	—

The matrix inversion of A gives the components of the vector V, that is, the nucleation probabilities for each limestone bed (the values $P_n(i)$, with $1 \leq i \leq m$). We thus obtain all the elements to carry out the computer simulation (Table 1 and Fig. 4). In a later step, an additional constraint accounting for average joint spacing is introduced. Note that because the joint spacing varies relatively little in a given limestone bed of the section considered, this average value simply reflects the periodicity of joints in this bed.

DATA COLLECTION

The numerical modelling requires input of the following parameters collected in the field.

(1) Statistical parameters:

m, the number of limestone beds ($m = 9$ in Fig. 7);

n_T, total number of joints belonging to the analysed set in the outcrop;

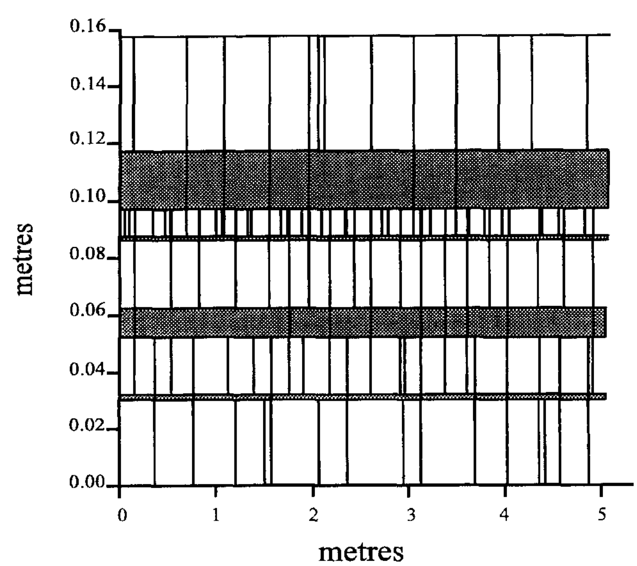


Fig. 4. Example of probabilistic modelling of extension joints in an imaginary vertical outcrop. Limestone beds are in white and mudstone beds are in grey.

$n_J(i)$, number of joints in limestone bed i, with $1 \leq i \leq m$;
$n_t(i, i+1)$, number of joints common to limestone beds i and $i+1$, with $1 \leq i \leq m-1$.

(2) Geometrical parameters:
$e(i)$, the thickness of limestone bed i, with $1 \leq i \leq m$;
$e'(i)$, the thickness of mudstone bed i below the limestone bed i, with $1 \leq i \leq m-1$;
L, the length of the outcrop;
L', the modelling outcrop length.

These parameters collected from a restricted area of the outcrop allow simulation of a larger outcrop or simulation of hidden outcrops, provided that the statistical and geometrical properties remain the same.

The numbers m, $e(i)$, $e'(i)$ and L define the outcrop geometry. With L' larger than L, a virtual modelling outcrop longer than the actual outcrop is created but with a similar joint distribution. The numbers $n_J(i)$ and $n_t(i, i+1)$ play an important role in the probabilistic approach because they allow calculation of propagation probabilities $P_p(i, j)$ in the program. The number n_T defines the number of joints to be nucleated inside the modelled outcrop.

The matrix equation (equation 4) is then solved and the unknown probabilities $P_n(i)$ are found. This set of probabilities only permits the geometrical reconstruction of the pattern, of course the set obtained is only one possible solution depending on the assumptions of the model. The product of $n_T \times P_n(i)$ gives the simulated number of joints nucleated in limestone bed i, hence the number of joints nucleated elsewhere and propagating into bed i is $n_J(i) - n_T P_n(i)$.

The field data show that for limestone bed i the periodicity is approximately given by $L/n_J(i)$, the ratio of the actual outcrop length to the number of joints in limestone bed i. The average joint spacing for each bed, calculated by the computer program, is taken as a critical constraint. But joint spacing has not to be considered as totally constant and, as observed in the field, it possesses significant dispersion which we assumed to fit a log-normal law. This modelling distribution is based on joint spacings measured in the field and provides realistic results in that it avoids production of artificial patterns showing the same statistical distributions. As stated earlier, joint spacing, and thus joint periodicity, depends indirectly on bed thickness. But an important contribution to periodicity perturbation and realistic sources of irregularity are joints propagating from other beds, a factor also accounted for in the model. Note that for joints propagating from one bed to another, there is an additional preference given to joints which do not propagate close to one already existing in the next bed. This additional condition simply reflects the reasonable mechanical inference that pre-existing joints dissipate joint propagation energy. In other words, joint propagation is less likely to occur into a bed at places where the stress has been released by earlier jointing. It is empirically introduced in the computational model by systematic calculation of the spacings between the joints expected to propagate into a given limestone bed and the joints already belonging to the limestone bed they propagated into. Then the joints that display the largest spacings are selected to be propagated. Note, incidentally, that such energetic requirements also account for additional geometrical characteristics that we ignored in the model, such as the partial coupling between joint periodicities in adjacent beds.

RESULTS

The following field data were collected throughout the Llantwit Major outcrop (Fig. 1): (1) thicknesses of limestone and mudstone beds; (2) numbers of joints going through mudstone beds into adjacent limestones, and numbers of joints per limestone bed; and (3) individual joint spacings within each limestone bed. Counting was performed along horizontal scan lines, parallel to bedding and perpendicular to the joints. The cross-section illustrated in Fig. 2 is approximately 2.5 m high and 10 m long.

The analysis of spacing data yielded statistical results which generally resemble those found in the literature from other exposures (Pineau, 1985; Huang and Angelier, 1989). The analysis provides evidence that a negative exponential function fits the cumulative distribution of the number of joints as a function of the number of beds crossed (Fig. 5a). The joint spacing distribution fits a log-normal distribution (Fig. 6a); spacing increasing with increasing bed thickness, although the range of available bed thicknesses does not allow definition of a function (Angelier *et al.*, 1989).

The outcrop at Llantwit Major was also used in the computer simulation (Table 2). It has nine limestone beds, 0.09–0.45 m thick, and eight mudstone beds, approximately 0.01–0.15 m thick. In order to compare the results of our numerical modelling with the field data,

Table 2. Actual input parameters for modelling, data from selected outcrops at Llantwit Major (see also Figs 1 & 7). $m = 9$ (number of limestone beds); $L = L' = 11$ m (length of the outcrop and length of the modelling outcrop); $n_T = 168$ (total number of joints belonging to the analysed set in the outcrop); $n_J(i)$ (number of joints in limestone bed i); $n_t(i, i+1)$ (number of joints common to limestone beds i and $i+1$); $e(i)$ (thickness of limestone bed i); $e'(i)$ (thickness of mudstone bed i)

Bed reference number	Limestones $e(i)$ (cm)	Limestones $n_J(i)$	Limestones $n_t(i, i+1)$	Mudstones $e'(i)$ (cm)
1	18	51	26	7
2	29	43	31	4
3	17	39	35	0
4	20	46	30	0
5	45	39	31	0
6	16	57	23	14
7	9	58	34	0
8	26	40	18	8
9	38	31	—	—

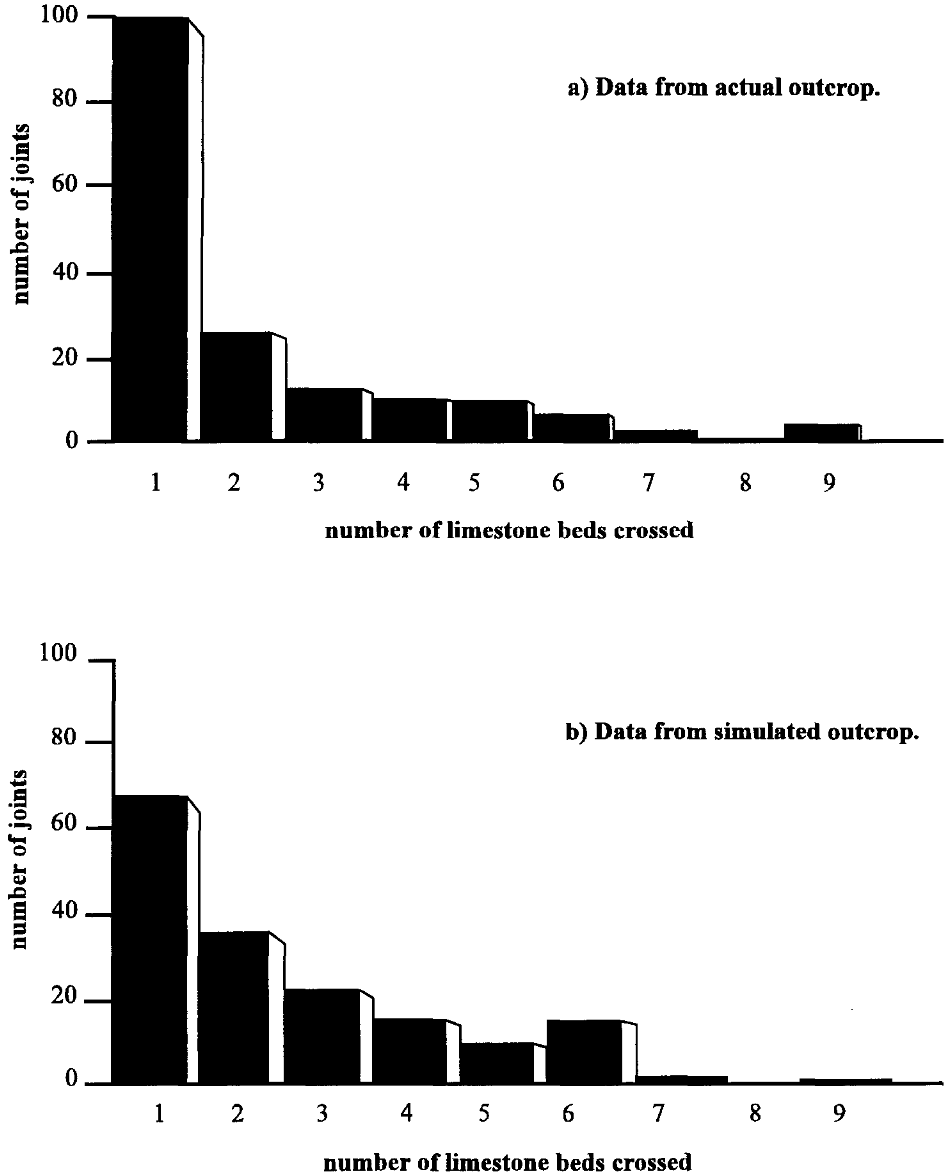


Fig. 5. Number of joints as a function of the number of beds crossed. (a) Actual case. (b) Modelled case. A negative exponential function fits both cases. Data from nine limestone beds at Llantwit Major.

we considered: (1) the simulated joint number distribution as a function of the number of limestone beds crossed (Fig. 5); and (2) the distribution of joint spacing (Fig. 6). It is important to fit the model of the actual and simulated vertical joint continuity, and the spacing/thickness ratio distribution (Table 3), to geological reality. In addition, model variations in spatial periodicity should fit the real variation (Fig. 7).

A histogram describes in a simple way the distribution of the number of joints vs the number of limestone beds crossed, such histograms for actual and simulated cases are easily compared (Fig. 5). Both approximately fit a negative exponential law. In more detail, Fig. 5 reveals minor but significant differences between the actual and simulated outcrop. In particular, the real data show more joints restricted to a single limestone bed, suggesting that joints born in a limestone bed and never propagated

Table 3. Means and standard deviations of the ratio joint spacing/bed thickness ($S(i)$ = joint spacing inside limestone bed i, $T(i)$ = thickness of bed i) for both the actual and simulated cross-sections. Data from nine limestone beds at Llantwit Major. The beds are numbered from top to base

Bed reference number	Bed thickness (m)	Average ratio $S(i)/T(i)$		Standard deviation	
		Outcrop	Model	Outcrop	Model
1	0.18	1.22	1.21	0.77	0.41
2	0.29	0.89	0.89	0.49	0.36
3	0.17	1.68	1.68	0.78	0.56
4	0.20	1.35	1.21	0.70	0.51
5	0.45	0.61	0.64	0.27	0.17
6	0.16	1.24	1.20	0.56	0.27
7	0.09	2.14	2.10	0.98	1.01
8	0.26	1.11	1.06	0.47	0.40
9	0.38	0.99	0.94	0.44	0.35
Cumulative data	—	1.29	1.26	0.79	0.68

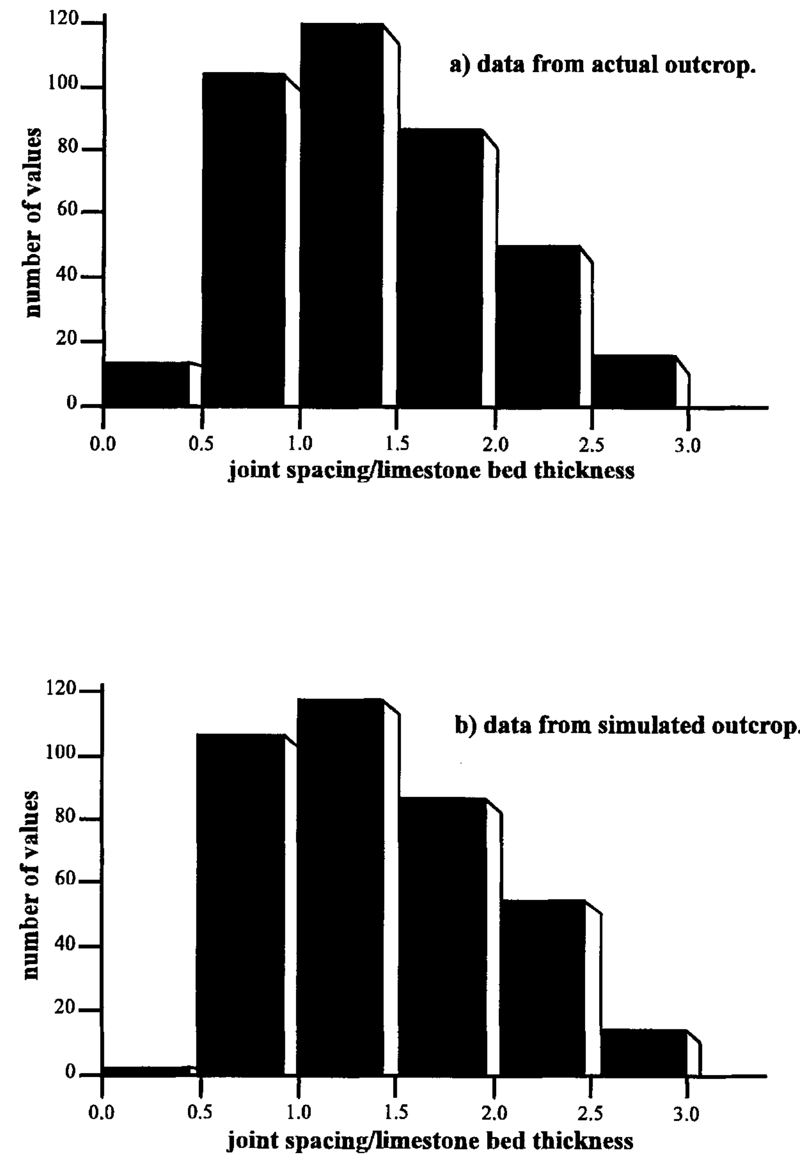


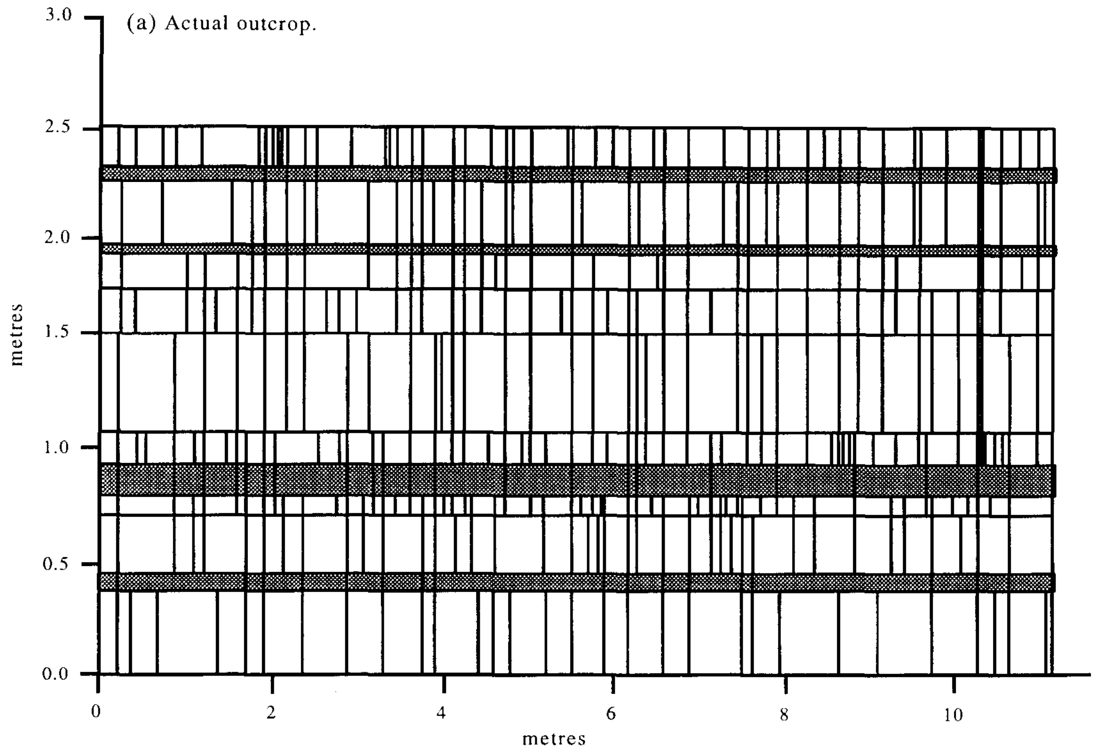
Fig. 6. Spacing/thickness ratio histograms. (a) Actual case. (b) Modelled case. Log-normal function fits both cases. Note that modelled standard deviation is less important in case (b) (see Table 3). Data from nine limestone beds at Llantwit Major.

beyond it are less numerous in the model than in the reality (Fig. 5, compare classes 1). However, examination of numerous results showed that this effect is principally accounted for by a systematic underestimation of the capacity for very high joints to develop (that is, joints continuously cutting many beds). This effect results from the lack of links between propagation probabilities at successive interbeds. A model which takes into account non-independent probabilities for joints to propagate across successive layers should result in more realistic patterns as far as the largest joints are considered. Other minor variations are explained either by uncertainties or by boundary effects (such as the histogram of real results shown in Fig. 5a, where all joints cutting more than eight beds are gathered in the ninth class because only nine beds were considered).

Finally, the two-dimensional probabilistic modelling allows reconstruction of a vertical cross-section perpendicular to the strike of the joints. Knowing that log-normal distribution of joint spacing is preserved in the model, it is then possible to compare the mean spacings and the standard deviations for each limestone bed (Table 3). This comparison reveals an acceptable fit in terms of mean spacings, with 5% as the largest misfit. However, the standard deviation computed from the model is generally smaller than the actual one; not surprisingly, this minor difference shows that the sources of dispersion remain somewhat larger in nature than in our numerical model. Along with consideration of the statistical quantitative parameters discussed above, one may also compare the real cross-section (Fig. 7a) with the computed one (Fig. 7b).

DISCUSSION AND CONCLUSIONS

The problem of the distribution of joints, especially in terms of spacing, has been addressed by statistical or



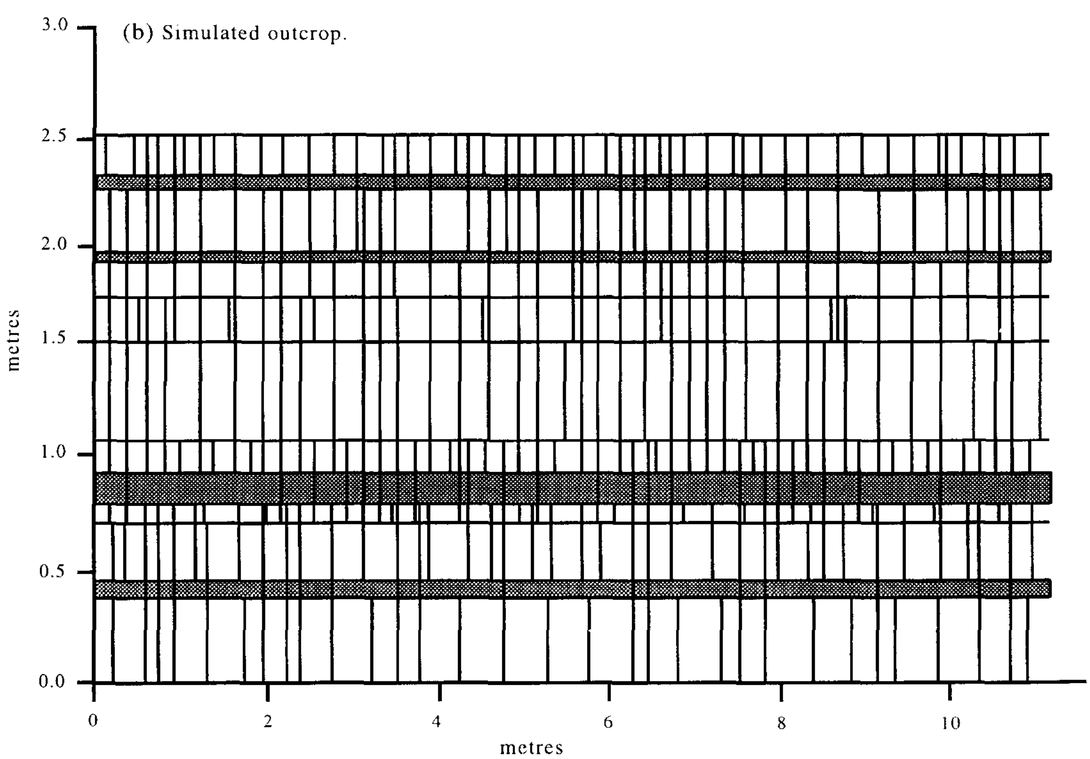


Fig. 7. (a) Natural studied outcrop at Llantwit Major. (b) Probabilistic simulation. Data from Llantwit Major. Limestone beds are in white and mudstone beds are in grey. Note that mean spacing of joints per bed is almost the same in both real and simulated cases (Table 3). However, spacings observed in case (b) are less scattered, and thus the standard deviation of spacing is smaller (Table 3).

mathematical means by Dershowitz and Einstein (1988) and Barthélémy (1992), and by mechanical analysis by Engelder (1987), Angelier *et al.* (1989) and Souffaché and Angelier (1989). The topic is important for determining fracture permeability in reservoirs (Wu and Pollard, 1991). It is also important because geologists use joint patterns for palaeostress reconstructions (Dyer, 1988; Bouroz, 1990; Hancock, 1994).

The probabilistic modelling of extension joint distribution provides an efficient tool because it allows reconstruction of fracture patterns based on only a few general geometrical parameters (e.g. the spacing distribution of fractures) and simple assumptions (e.g. the sense of vertical fracture propagation across bed interfaces). We addressed the common, albeit particular, case of a set of vertical joints in a horizontally layered rock mass. Many geological reservoirs comprising alternating competent and incompetent beds are cut by such joints. Our reconstruction of the joint pattern is controlled by the probability of each joint propagating into the next competent bed (in our case limestone) when it reaches the common boundary represented by an incompetent bed (i.e. mudstone).

In order to check the validity of the probabilistic modelling, we compared actual and simulated patterns (Fig. 7). It is important to realize that our data collection did not aim at simply providing the parameters indispensable for building the model, which are few, but it also aimed at verifying whether some of the assumptions were correct, such as the log-normal dispersion law, thus allowing a thorough statistical comparison between observed and reconstructed joint patterns.

It is important to distinguish between the input of geological parameters in our modelling, based on few major values listed earlier, and the comparison between real and simulated cross-sections, which require detailed examination of geometrical patterns (Fig. 7). Of course, some major general parameters resulting from modelling deserve careful consideration because they depend on the rock mass: this is the case for the number of joints per competent bed and the number of joints crossing each incompetent bed. Despite simplifying assumptions, our modelling experiments resulted in satisfactory reconstructions in that the most important statistical properties fitted both the actual and simulated fracture patterns. In detail, the reconstruction displays higher levels of regularity (Fig. 7). Among the basic statistical laws which fitted the real patterns studied are the log-normal distribution of fracture spacings, joint spacing increasing with bed thickness and the negative exponential distribution of joint numbers as a function of the number of beds crossed.

The probabilistic modelling followed here has the capacity to provide realistic reconstructions of joint patterns provided that a limited number of parameters (listed above) is known. However, the limitations of the modelling should be kept in mind. First, it deals with a two-dimensional case: this is a reasonable approximation where one set of joints is dominant, but would not be appropriate where several sets are well represented. Second, we analysed joints that are perpendicular to bedding within a layered rock sequence comprising alternating limestones and mudstones; our analysis would also be applicable to other layered sequences. Such patterns are common and allow the introduction of simplifying assumptions as discussed before. However, application to massive or irregularly layered rock units, or to joints oblique to layering, would require different modelling. We point out that in such cases, because sources of variations are numerous, our model should not be applied using only a few parameters as constraints; it should involve more detailed statistical control by means of field or sub-surface observations.

Some field parameters may be difficult to obtain, for example joint height distribution, which influences fracture permeability through continuity and connectivity. Where the height of control outcrops is limited, or where there are only sub-surface data, the importance of joints with a large vertical extent cannot be rigorously estimated. From a qualitative point of view, our field observations showed that larger joints are commonly composite joints (Helgeson and Aydin, 1991). This highlights the difficulty of rigorously defining the vertical extent of such large joints. This geometrical aspect was not taken into consideration in our probabilistic modelling.

Provided that the requirements listed above are met, our probabilistic modelling provides a powerful tool for reconstructing joint patterns in the absence of strong constraints from direct observation. The theoretical analysis and the case study presented highlight this potential because knowledge of outcrops allow detailed checking of the results (Fig. 7). Starting from a geometrical and statistical analysis of an outcrop of limited size, we are able to simulate these properties in hidden outcrops, provided that they are likely to be of similar geometry and lithology. We thus consider this work as a preliminary step in the preparation of more complex, three-dimensional multiset models.

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